Distribution of the Sample Proportion

We selected Q8.2.16 (p.385) as an example of using StatCrunch to calculate probability of $p$.

Are You Satisfied? According to a study done by the Gallup organization, the proportion of Americans who are satisfied with the way things are going in their lives is 0.82.

(a) Suppose a random sample of 100 Americans is asked, “Are you satisfied with the way things are going in your life?” Describe the sampling distribution of $p$, the proportion of Americans who are satisfied with the way things are going in their life. Be sure to verify the model requirements.

(b) In the sample obtained in part (a), what is the probability the proportion who are satisfied with the way things are going in their life exceeds 0.85?

(c) Would it be unusual for a survey of 100 Americans to reveal that 75 or fewer are satisfied with the way things are going in their life? Why?

(a) Suppose a random sample of 100 Americans is asked, “Are you satisfied with the way things are going in your life?” Describe the sampling distribution of $p$, the proportion of Americans who are satisfied with the way things are going in their life. Be sure to verify the model requirements.

$$n = 100, \ p = 0.82$$

Since $np(1-p)=100(0.82)(1-0.82)=14.76 \geq 10$, the shape of the sampling distribution of $p$ is approximately normal.

The mean of $p$ is $\mu_p$ and $\mu_p = p = 0.82$.

The standard deviation of $p$ is $\sigma_p$ and $\sigma_p = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.82(1-0.82)}{100}} = 0.038418745 \approx 0.038$. 
(b) In the sample obtained in part (a), what is the probability the proportion who are satisfied with the way things are going in their life exceeds 0.85?

\[ P(p > 0.85) \]

\( p \) is normally distributed with \( \mu_p = 0.82 \) and \( \sigma_p = 0.038 \)

Step 1: Log onto StatCrunch and get a blank data sheet.

Step 2: Click Stat \( \rightarrow \) Calculators \( \rightarrow \) Normal.

Step 3: 1) When the normal distribution dialogue box pops up. Click the Standard tab.

2) For this \( p \) variable, Mean=0.82 for \( \mu_p \), and Std. Dev. =0.038418745 for \( \sigma_p \).

3) Use \( \triangledown \) to select \( \geq \) \( \rightarrow \) Input 0.85

4) Click Compute.
The shaded area represents \( P(p \geq 0.85) \).

\[
P(p \geq 0.85) = 0.21743983 \approx 0.2174.
\]

Interpretation:
The probability that a random sample of \( n = 100 \) Americans results in at least 85% are satisfied with with their life is approximately 0.22. This means that about 22 out of 100 random samples of size 100 will result in at least 85% are satisfied with their life if the population proportion of Americans who are satisfied with their life is 0.82.

(c) Would it be unusual for a survey of 100 Americans to reveal that 75 or fewer are satisfied with the way things are going in their life? Why?

\[
n = 100, x = 75, \quad p = \frac{x}{n} = \frac{75}{100} = 0.75
\]

--> Find \( P(p \leq 0.75) \).

\( p \) is normally distributed with \( \mu_p = 0.82 \) and \( \sigma_p = 0.038418745 \).

Use the same normal distribution dialogue box with Mean: 0.82 and Std. Dev.: 0.038418745.

Step 1: 1) Use \( \nabla \) to select \( \leq \) → Input 0.75.

2) Click Compute.
The shaded area represents \( P(p \leq 0.75) \).

\[
P(p \leq 0.75) = 0.03422543 \approx 0.0342.
\]

Interpretation:
The probability that a random sample of \( n = 100 \) Americans results in at most 75% are satisfied with their life is approximately 0.03. This means that about 3 out of 100 random samples of size 100 will result in at most 75% are satisfied with their life if the population proportion of Americans who are satisfied with their life is 0.82.

Since \( P(p \leq 0.75) \) is 3 out of 100, it is unusual for a survey of 100 Americans to reveal that 75 or fewer are satisfied with the way things are going in their life.